JOURNAL OF ENGINEERING AND MANAGEMENT IN INDUSTRIAL SYSTEM VOL. 11 NO. 2 YEAR 2023
e-ISSN 2477-6025
https://doi.org/10.21776/ub.jemis.2023.011.02.6

A DECISION-ANALYTIC, SIMULATION BASED MANAGEMENT MODEL FOR A TWO-SHIFTS QUEUE SYSTEM WITH CUSTOMER RENEGING

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Abstract This paper shows a management model for a queuing system operating two shifts and where client reneging is present. The model provides the optimal active server policy considering lost clients and cost. Using influence diagrams and decision trees, the proposed model implements a Bayesian scheme to update the initial knowledge in response to the observed number of lost customers in the first shift. In addition, a utility function is used to represent the preferences of the decision-maker and simulation to determine the consequences and probabilities of the trees. The results are displayed as the number of active servers recommended for each shift, according to the preferences of the decision-maker and the clients lost in the first shift. For the case study, it is found that if the weight for the number of lost clients in the utility function (k_{NL}) is less than 0.63, a single server should be used in both shifts, while when k_{NL} is greater than said value, the first shift opens one server, while a second one is activated for the next shift depending on the number of lost clients in the first shift.

Keywords: Queue Theory, Simulation, Decision trees, Influence Diagram, Customer Reneging

1. Introduction

Queue systems arise in service facilities in which customers should wait in line while the servers are busy. In systems with reneging, customers are only willing to wait for service a certain amount of time, after which they leave the facility [1]. If, additionally, the system operates several shifts, its management will require to decide how many servers to have active each shift.

A mathematical model to support the management of queuing systems with customer reneging and that operates several shifts should consider two essential qualities of these systems. The first, present in the management of queues in general, is the trade-off between operating costs and customer satisfaction. The second one is more subtle and specific to these systems, being given by the change of information that occurs as the number of customers lost in a shift becomes known. This information change should be quantified and taken advantage of when deciding the number of servers to activate in subsequent shifts.

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Published online at http://Jemis.ub.ac.id

This manuscript shows the development of the mathematical model for managing a case study of a queueing system with client reneging and operating two shifts. Developed from a perspective and with tools of Decision Analysis [2], the model includes the particularities of the administration of these systems: a utility function provides for the trade-off between costs and the number of lost customers while an influence diagram and its corresponding decision tree are used to represent the structure of the decisions, allowing the Bayesian updating of previous knowledge based on the observation of the number of customers lost in the first shift.

There are several recent reports on the use of queuing theory in service systems administration. In 2022, Alnowibet et al. [3] reported the optimization of the operation of the airport at El Cairo, while the paper of Shone et al. [4], appearing in 2021, describes other applications of said theory to airport management; Tarakci et al. [5] address hospital management, balancing costs and diagnostic quality objectives and Kerbache and MacGregor-Smith [6] use networks of open finite queues for route optimizing in

Cite this Article As Hernandez, Mario Luis Chew et al (2024). A Decision-Analytic, Simulation Based Management Model For A Two-Shifts Queue System With Customer Reneging. Journal of Engineering and Management in Industrial System, 11(2), p145-152
Paper Submitted : Sept, 22th 2023
Paper Published : Jan, 10th 2024

e-ISSN 2477-6025
https://doi.org/10.21776/ub.jemis.2023.011.02.6

setting the location and layout of a manufacturing facility.

Fewer reports deal with managing waiting lines in which customers can leave the queue, with Pala and Zhuang [7] and Wang and Zhuang [8] applying game theory to consider the strategic behavior of terrorists or smugglers in the security check points of visa offices or airports, aiming to strike a balance between the loss of regular customers, security and waiting time, while Trigos et al. [9] use simulation to consider customer reneging in the management of a system of complex servers, with the objective of improving service quality and reducing costs.

Queuing theory offers widely known formulas for system performance metrics for queues in which customer reneging is absent and interarrival and service times are exponentially distributed [1]. However, this it is not the case of systems with customer reneging and/or non-exponential times, whose complexity makes them the object of the current efforts of various research groups. Formulas for the performance metrics of systems with exponential abandonment times are included in Wang and Bin [10] and Sasanuma and Scheller-Wolf [11]. Nasrallah [12] studies the impact of priority policies on performance metrics; Ammar et al. [13] apply generating functions to obtain the probability density function of the number of customers in the queue and Xiong et al. [14] use Volterra integrals to analyze queues with deterministic reneging times. Finally, Wu and Ke [15], Dimou et al. [16] and Economou et al. [17] address systems where servers go into idle periods, affecting customer reneging behavior and Pazgal and Radas [18] describe an empirical study to validate predictions about the actual behavior of customers deserting the queue.

Decision Analysis (DA), as envisioned by its founder R.A. Howard [19] is a discipline that aims to help difficult decision making, making the most of the available information. By following the recommendations derived by a DA approach to a problem, the stakeholder is guaranteed to abide to a set of axioms of rational decision making [20]. The reviewed managing models of queue systems with customer reneging, described in the second paragraph of this section, neither take a DA perspective to model the problem nor consider the gain of information that occurs by observing the customers leaving the queue, as done here through a Bayesian scheme implemented with decision trees and influence diagrams. Finally, as formulae for these systems performance metrics haven't been reported in the literature, said metrics are determined here using process simulation.

2. Methods

2.1 Case study description

The case study is shown in Figure 1. The facility has a number of servers in parallel, which can be either open (having a worker assigned to the server) or closed.

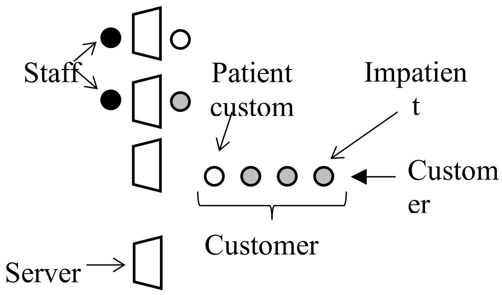

Fig 1 Case study

Customer reneging behavior is modeled by dividing customers into "patient" or "impatient" based on how long they are willing to wait in line before leaving the system. The system operates two shifts, between which the number of open servers can be adjusted.

2.2 Process simulation

Discrete event simulation [21] was used to determine the probability distribution of the number of customers leaving the queue in a working shift (N_L), conditional on the proportion of impatient clients (X_I) and the number of active servers in the shift (D).

2.3 Decision structure

An influence diagram, in which rectangles indicate decisions and ovals uncertainties, represents the decision structure. In these diagrams, if an arrow reaches a rectangle, it means that the decision is made knowing what happened in the figure

e-ISSN 2477-6025
https://doi.org/10.21776/ub.jemis.2023.011.02.6

(node) originating the arrow. On the other hand, if an arrow reaches an oval, it indicates that the probability distribution of the variable in the oval depends on the result of the node emanating the arrow [2]. Figure 2 shows the decision of the number of servers active for the first shift, where brackets indicate probability distributions.

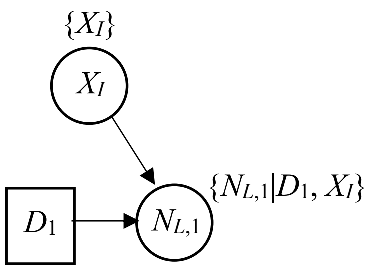

Fig 2 Diagram for deciding the number of active servers in first shift

{X_I} is the a-priori (i.e., before the start of the first shift) probability distribution of X_I. The distribution {$N_{L,1}|D_1, X_I$}, where $N_{L,1}$ is the number of clients lost in the first shift and D_1 the number of active servers in this shift, is calculated using simulation. At the start of the second shift, the number of servers to open (D_2) is decided after $N_{L,1}$ becomes known.

The probability distribution of the number of lost customers in the second shift, $N_{L,2}$, depends on D_2 and X_I. However, $N_{L,1}$ is relevant to X_I (i.e., knowing how many customers left the queue in the first shift gives information about the proportion of them that are impatient) so the decision of how many servers to activate for the second shift is made with more information. This knowledge update can be represented by reversing the arrow between X_I and $N_{L,1}$ in figure 2 (figure 3).

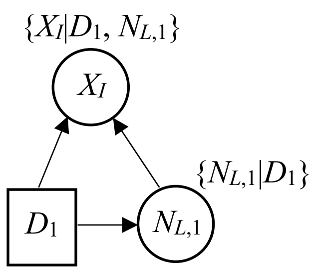

Fig 3 Reversal of the arrow between X_I and $N_{L,1}$ in figure 2

The distribution of $N_{L,1}$ conditional on D_1 is obtained from {$N_{L,1}|D_1, X_I$} and {X_I} through the formula for calculating probabilities by conditioning over a partition of values of X_I, while {$X_I|D_1, N_{L,1}$} is derived from {$N_{L,1}|D_1, X_I$} using Bayes' theorem. The influence diagram for the two shifts decision is shown in figure 4. This diagram shows that the decision of the servers to keep open for the second shift (D_2) is made after seeing $N_{L,1}$. The diagram is completed with a value node (hexagon) with the important consequences for the decision, that are the total number of lost customers $N_{L,1}+ N_{L,2}$ and the cost $C(D_1, D_2)$ of the active servers

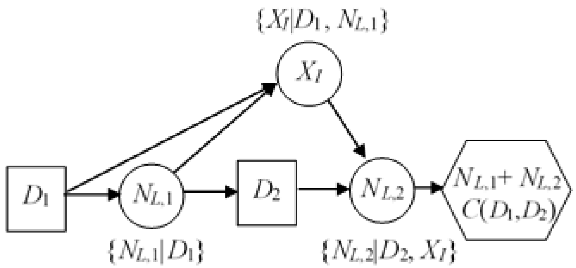

Fig 4 Influence diagram tor the complete problem

To evaluate the influence diagram, it needs to be translated into a decision tree. In order to do so, each variable has to be discretized into a set of possibilities (e.g., D_1 can take the value of D_1^1, D_1^2,..., or D_1^{nd1}, with nd_1 being the number of D_1 possibilities). A schematic diagram of the tree is shown in figure 5, where *nd*1, *nnl*1, *nd*2, *nxi*, and *nnl2* are the number of possible values of, respectively, D_1, $N_{L,1}$, D_2, X_I, and $N_{L,2}$. In these trees, uncertainties appear as circles, while decisions are represented by squares. The lines emanating from a decision represent alternatives, while those radiating from uncertainties represent its possibilities, with the relevant probabilities written below each line. At the tips of the tree appear the consequences, which consist of the total number of lost customers ($N_{L,1}$+$N_{L,2}$) and the cost $C(D_1, D_2)$, given by the number of servers active in the two shifts.

The simulation model provides the probabilities $P(N_{L,1}^i|D_1^j, X_I^k)$ and $P(N_{L,2}^i|D_2^j, X_I^k)$. Given the a-priori probability distribution $P(X_I^k)$, probabilities $P(N_{L,1}^i|D_1^j)$ are calculated as

e-ISSN 2477-6025
https://doi.org/10.21776/ub.jemis.2023.011.02.6

$$P\left(N_{L,1}^{i}|D_{1}^{j}\right) = \sum_k P\left(N_{L,1}^{i}|D_{1}^{j},X_{I}^{k}\right) \times P\left(X_{I}^{k}\right) \quad (1)$$

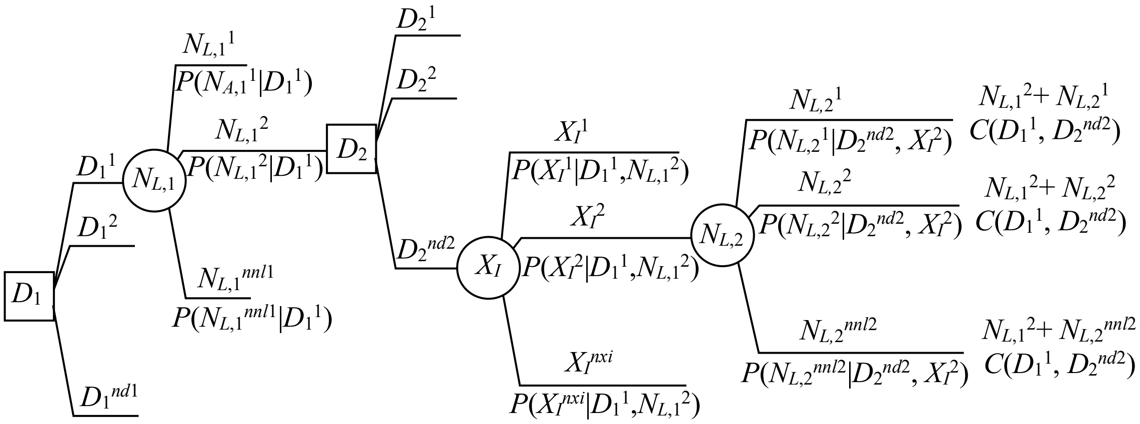

Fig 5 Decision tree for the number of active servers in both shifts

While probabilities $P(X_I^k|D_1^j, N_{L,1}^i)$ are calculated from Bayes´ theorem

$$P\left(X_{I}^{k}|D_{1}^{j},N_{L,1}^{i}\right) = \frac{P\left(N_{L,1}^{i}|D_{1}^{j},X_{I}^{k}\right)}{P\left(N_{L,1}^{i}|D_{1}^{j}\right)} P\left(X_{I}^{k}\right) \quad (2)$$

Where the dependence between X_I^k on D_1^j, is omitted, as, by independence $P(X_I^k|D_1^j)=P(X_I^k)$.

2.4 Value model

The consequences, total number of lost customers $N_{L,T}=N_{L,1}+N_{L,2}$ and cost C, are given a preference metric through an additive utility function [22].

$$U(N_{L,T}, C)=k_{NL}U_{NL}(N_{L,T})+k_C U_C(C) \quad (3)$$

The weights k_{NL} and k_C add to one. The one-dimensional utility function U_{NL} takes the values of one and zero, respectively, when the number of total lost customers is zero and its highest possible value for any of the alternatives. Correspondingly, the cost utility function, U_C, is one when using one open server per shift and zero for the alternative using the maximum number of open servers. When solving a decision tree, the consequences at the tree tips are replaced by their utilities, and the tree is "folded" from right to left. An uncertainty is calculated as the expected value of the utilities of the branches emanating from it, while the utility of a decision is taken as that of the alternative with the highest utility that stem from its rectangle.

3. Results and discussion

To provide numerical results, the time between customer arrivals is assumed to be exponentially distributed with mean 10 min, while the service times are taken as uniformly distributed between 15 and 25 min. The maximum waiting time of a patient customer is assumed to be uniformly distributed between 30 and 50 min, while that of an impatient client is taken as a uniform variable between 10 and 30 min. A single shift is taken as 300 minutes long. Simulation is used to obtain the probability distribution of the number of customers lost in a shift (N_L) given the proportion of clients that are impatient X_I and the number of servers active in the shift D. For example, if X_I=0.333 (that is, one third of the clients are impatient) and only one server is active (D=1) the probability distribution of N_L, calculated over 500 replications, is shown in figure 6.

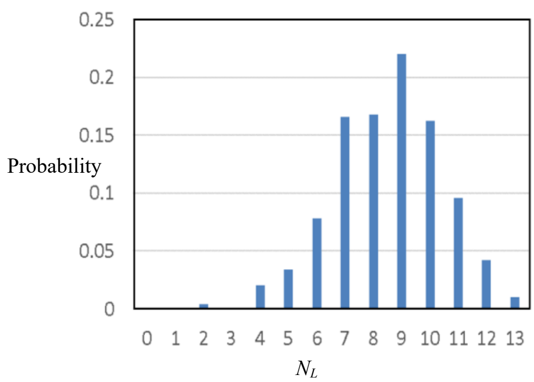

Fig 6 Probability distribution $P(N_L|D=1, X_I=0.333)$

e-ISSN 2477-6025
https://doi.org/10.21776/ub.jemis.2023.011.02.6

3.1 A-priori knowledge

The proportion of impatient clients X_I is discretized into the values "High" (X_I=2/3) and "Low" (X_I=1/3). The probability of the proportion of impatient customers being high is denoted $P(X_I{}^H)=P(X_I=2/3)$. The a-priori state of knowledge, before starting the first shift, is taken as $P(X_I{}^H)$=0.5, that is, it is thought that the proportion of impatient clients is equally likely to be high or low.

3.2 A-posteriori knowledge

At the end of the first shift, that operated with D_1 servers active, an observation of $N_{L,1}$, the number of customers lost in that shift, becomes available. So the probability that the proportion of impatient clients is high changes from $P(X_I^H)$=0.5 to $P(X_I^H|D_1,N_{L,1})$. These "a-posteriori" probabilities are shown in figure 7.

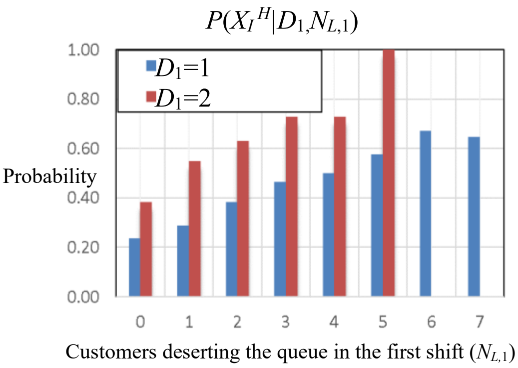

Fig 7 Probability of X_I high, conditional on the number of active servers and lost customers in the first shift

Figure 7 shows how the knowledge of X_I is changed by the observation of $N_{L,1}$. For example, if the first shift opens a single server (D_1=1) and no clients are lost in this shift, the probability of X_I being high drops from 0.5 to $P(X_I{}^H|1, 0)$=0.22 (indicating an increased belief that X_I is low). If one server is active in the first shift and 5, 6, or 7 customers desert from the queue in this shift, $P(X_I^H|1, N_{L,1})$ takes a value of 0.57, 0.67 and 0.66 respectively, indicating that the confidence of X_I being high increases slightly from it's a-priori 0.5 level.

If the first shift has two servers active (D_1=2) and no customers are lost, the updated probability of X_I to be high, $P(X_I^H|2,0)$ is 0.41, slightly down from its a-priori value of 0.5. Under a two active server operation of the first shift, if 3 or 4 clients are lost, $P(X_I^H|2, N_{L,1})$ is calculated as 0.73 and 0.78 respectively, while if 5 clients are lost, this gives certainty that the proportion of impatient clients is high, $P(X_I^H|2,5)=1$. It is observed that when operating the first shift with two active servers instead of one, the observation of $N_{L,1}$ is more informative (it produces greater changes in the probability of X_I with respect to its a-priori value).

3.3 Value parameters

The utility function $U_{NL}(N_{L,T})$ on the total number of lost customers $N_{L,T}$ was taken as a quadratic function through the points $U_{NL}(0)$=1, $U_{NL}(7)$=0.8 and $U_{NL}(14)$=0, as the highest value of lost customers that can occur in a single shift is 7, giving a maximum of 14 for two shifts. This shape of $U_{NL}(N_{L,T})$ penalizes large $N_{L,T}$ values over small ones (figure 8).

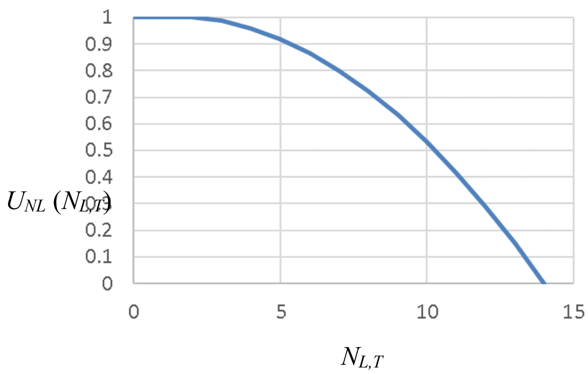

Fig 8 One dimensional utility function for the number of customers that left the queue

The cost of operation depends on the total number of active servers in both shifts, $D_T=D_1+D_2$, so the cost utility function $U_C(D_T)$ equals one when using one active server in both shifts, (D_T=2), zero when using two active servers in both shifts (D_T=4) and 0.5 if one shift opens a single server and the other opens two (D_T=3).

The weights k_{NL} and k_C of the overall utility function (3) measure the manager's relative preferences for lost customers and number of active servers, and are subjective by nature, meaning that they depend on the actual person making the decision [22]. The values of these weights are derived by having the stakeholder compare transitions between the balanced dimensions (performance metrics). In the present context, the transitions between the maximum and minimum values of the performance metrics are denoted $\Delta N_{L,T}$ and ΔD_T,

e-ISSN 2477-6025
https://doi.org/10.21776/ub.jemis.2023.011.02.6

characterized as $\Delta N_{L,T}$ ="*change from 14 to 0 lost customers*" and ΔD_T ="*change from 4 to 2 active servers*". The values of k_{NL} and k_C are derived from the manager's relative preference between $\Delta N_{L,T}$ and ΔD_T. If, for example, the manager prefers to have $\Delta N_{L,T}$ instead of ΔD_T, this means that $k_{NL}>k_C$ and, were the manager indifferent between the two transitions, $k_{NL}=k_C=0.5$.

3.4 Model recommendations

The model results are shown in figure 9, for different values of k_{NL} (k_C is the complement to 1). Different colors are used to indicate the number of servers to activate in the first shift, D_1, and in the second shift D_2.

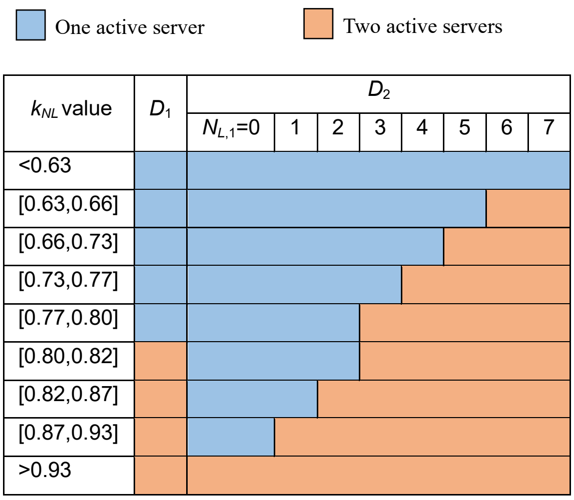

Fig 9 Recommended number of active servers for both shifts

The color of the column "D_1" indicates how many servers should be active in the first shift, while the number of servers to be open in the second shift is indicated by the colors of the seven columns under the "D_2" header, and depends on the number of customer that deserted the queue in the first shift, $N_{L,1}$. For example, if k_{NL}<0.63, both the first and second shifts will operate with a single server active, regardless of the value of $N_{L,1}$. If 0.73<k_{NL}<0.77, only one server should be active in the first shift, policy that is maintained in the second shift if less than four customers are lost in the first shift, otherwise a second server should be activated for the second shift. Similarly, if 0.80<k_{NL}<0.82, two servers are active in the first shift and, if fewer than three clients leave the queue in this shift, one server closes for the second shift, otherwise the two servers are kept active for the second shift.

The results indicate that, the greater the value of k_{NL}, the less stringent the conditions for which the usage of two servers is recommended. As k_{NL} represents the importance of the number of lost customers relative to the operation costs, if k_{NL} grows, the manager prefers to open more servers (incurring more operational cost) in order to minimize the number of customers leaving the queue.

4. Conclusion

The discipline of Decision Analysis, as introduced by R.A.Howard [23], seeks to support difficult decision making, among which are situations with uncertainty and multiple objectives. This manuscript has shown, through the detailed analysis of a specific case study, the application of two of its tools for decision structuring (influence diagrams and decision trees) and one for preference representation (utility function) to the problem of the administration of the case study of a system of queues with customer reneging, operating several shifts between which servers can be closed or opened.

The model developed here incorporates, through a Bayesian update of previous probability distributions, the change of information caused as the number of customer desertions in the first shift is observed. This is a distinct feature that a management model for this type of problems should consider, and that isn´t covered by previously reported research.

In the current, highly competitive business environment, customers generally have several businesses at hand that can provide a service. This wide availability means that the customer, having initially joined the waiting line of one business, can shift to a different supplier if the wait time grows excessive. An example of this situation occurs in bus terminals or airports, where the booths of competing transport companies are located next to each other, and customers can easily leave one company line to join the line of another. The application of the methodology illustrated in this article could represent a competitive advantage for one of such transport companies.

The implementation of the model presented requires knowing how many customer desertions have occurred in a shift. This information could be observed directly or captured by cameras equipped with image

e-ISSN 2477-6025
https://doi.org/10.21776/ub.jemis.2023.011.02.6

recognition. The model would be solved by means of a computer program fed with the observed number of desertions, and the recommendation would then be transmitted to the chief of staff, so to implement the necessary changes in the number of active servers.

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